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Combined forced and natural convection heat transfer from a horizontal cylinder embedded in a porous medium

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Abstract—This paper reports the results of an experimental study of heat transfer by combined forced and natural convection from a horizontal cylinder embedded in a porous medium composed of randomly packed glass spheres saturated with water. The direction of the flow of water was horizontal and normal to the longitudinal axis of the cylinder. The diameter of the cylinder, D, was 11.45 mm and the equivalent diameter of the glass spheres was 3.072 mm. It is shown that the condition $Gr_K/Re_D^2 \leqslant 0.5$ represents a conservative criterion for segregating heat transfer data that are predominantly governed by forced convection from those in which natural convection effects are significant. A correlation hypothesis for convection heat transfer which is based upon four assumptions, primary among which is that the flow can be (conceptually) regarded as being composed of 'coarse' and 'fine' components, is presented. This hypothesis is shown to provide a basis for successfully correlating a set of experimental heat transfer data that extends from the Darcy regime into the turbulent regime and spans the intervening Forchheimer and transition regimes. It is suggested that the correlation procedure adopted here may yield useful results if applied to other geometries such as, for example, forced convection heat transfer in ducts packed with porous media.

INTRODUCTION

A THEORETICAL solution of the complex general problem of forced convection heat transfer from a horizontal cylinder embedded in a porous medium with cross flow has not yet been achieved. We present herein the results of an experimental study of this problem in which the porous medium consists of randomly packed glass spheres. These results include a set of empirical correlation equations that are based upon a hypothesis which makes use of information previously published in refs. [1–3]. The relevant information contained in these references is cited in the review of the literature which follows.

REVIEW OF THE LITERATURE

Since there is no sharp dividing line between natural and forced convection, it has been found helpful to define a criterion whereby natural convection effects may be neglected relative to the other. Fand and Keswani [1] have published the results of an experimental investigation of combined natural and forced convection heat transfer from a horizontal cylinder to water with cross flow which shows that when

$$Gr_D\,Re_D^{-2} \leqslant 0.5 \tag{1}$$

the predominant heat transfer mechanism is forced convection. When forced convection is predominant, Fand and Keswani found that the following heat transfer correlation equation applies to the case of a horizontal cylinder in cross flow

$$Nu = (0.255+0.699Re_D^{0.5})Pr^{0.29}. \tag{2}$$

In equation (2) all fluid properties are evaluated at the mean film temperature.

Fand and Keswani [1] have shown that for values of $Gr_D\,Re_D^{-2}$ between 0.5 and 4 the contribution of natural convection effects of the Nusselt number for a horizontal cylinder in cross flow can be accounted for by adding a natural convection term to equation (2). For horizontal cross flow, which is the particular geometry considered herein, the added natural convection term is directly proportional to $Gr_D\,Re_D^{-2}$.

Fand *et al.* [2] have determined a correlation equation for natural convection heat transfer from a horizontal cylinder embedded in a porous medium that is relevant to the present study. Their equation includes a term which accounts for viscous dissipation in high viscosity fluids such as silicone oil. If viscous dissipation is negligible, as in the present study using water, the said correlation equation reduces to the following:

$$Nu = 0.653Gr_K^{0.649}\,Pr^{0.525}. \tag{3}$$

In order to deal with the heat transfer problem under consideration, it is necessary to have certain information concerning the flow of fluids through porous media. Reference [3] discusses five regimes of flow through porous media that are relevant to the present study, namely, the Darcy (for which $Re_{DL} < Re_d \leqslant Re_{DH} = 2.3$)† Forchheimer (for which

† The lower bound for the Darcy regime, Re_{DL}, is less than 10^{-5}. The cited upper bound, $Re_{DH} = 2.3$, is correct here but is reported incorrectly to be 2.1 in Table 1 in ref. [2].

NOMENCLATURE

A, B, C	numerical constant
C_p	specific heat of fluid at constant pressure
D	diameter of test cylinder; also distance from leading edge of a flat plate
d	equivalent diameter of glass spheres
Gr_D	Grashof number, $gD^3\beta\Delta T/v^2$
Gr_K	Grashof number, $gKD\beta\Delta T/v^2$
g	gravitational constant
h	heat transfer coefficient
K	permeability
k	effective thermal conductivity per equation (5)
k_e	equivalent thermal conductivity
k_f, k_s	thermal conductivity of fluid, solid
N	number of data points
Nu	Nusselt number, hD/k
P'	negative of the pressure gradient
Pr	Prandtl number, $C_p\mu/k$
q	heat transfer per unit area per unit time
Re_D	Reynolds number, Dv/ν
Re_d	Reynolds number, dv/ν
S_0	surface area per unit volume of porous medium
T_b, T_s	bulk, surface temperature
ΔT	temperature difference, $(T_s - T_b)$
v	magnitude of Darcian velocity (volume rate of flow per unit area).

Greek symbols

α	a function of ε, $(1-\varepsilon)^2/\varepsilon^3$
β	coefficient of volumetric expansion of fluid; also a function of ε, $(1-\varepsilon)/\varepsilon^3$
ε	porosity
ε_w	wall-corrected porosity per equation (4)
κ	Kozeny–Carman constant
μ	dynamic viscosity of fluid
ν	kinematic viscosity, μ/ρ
ρ	density of fluid
ϕ	a numerical parameter.

Subscripts

cal	a quantity determined by calculation
exp	a quantity determined by experiment.

Error notation

E	percent error, $100(Nu_{exp} - Nu_{cal})/Nu_{exp}$
E_{max}	maximum percent error
E_{md}	percent mean deviation of error, $\sum_{i=1}^{N} \lvert E_i \rvert / N$
E_{rms}	percent root mean square error, $\left[\sum_{i=1}^{N} E_i^2\right]^{0.5} \Big/ N$.

$5 = Re_{FL} < Re_d \leqslant Re_{FH} = 80$) and turbulent (for which $Re_d > Re_{TL} = 120$) regimes plus the transitions from Darcy to Forchheimer flow and from Forchheimer to turbulent flow.

In the Darcy regime the flow is dominated by viscous forces and it is laminar and the following equation applies:

$$\frac{P'd}{\mu v} = Cd \tag{4}$$

where the constant $C = \kappa S_0^2 \alpha$ and $\alpha = (1-\varepsilon)^2/\varepsilon^3$. For porous matrices composed of spheres $S_0 = 6/d$. The inverse of C is called the permeability, denoted by K, of the porous medium. The value of the Kozeny–Carmen constant κ is reported to be 5.34 in ref. [3].

In the Forchheimer regime the flow is still laminar but inertial effects become significant and the following equation applies:

$$\frac{P'd}{\mu v} = C_1 d + C_2 Re_d \tag{5}$$

where $C_1 d = A\alpha/d$, $C_2 = B\beta/d$ and $\beta = (1-\varepsilon)/\varepsilon^3$. The values of the dimensionless constants A and B are reported in ref. [3] to be as follows: $A = 182$, $B = 1.92$. It is shown in ref. [3] that equation (5) also applies in the turbulent flow regime but that the values of the dimensionless constants (primed to differentiate their notation) are as follows: $A' = 225$, $B' = 1.61$.

The transitions between Darcy and Forchheimer flow and between Forchheimer and turbulent flow are difficult to characterize because they are not representable by simple equations such as equations (4) and (5). It is suggested in ref. [3] that this difficulty can be overcome without incurring significant error by assuming that a fictitious 'transition Re_d' exists, denoted by Re_{DF} ($Re_{DF} = 3$), at which the flow abruptly changes from Darcy to Forchheimer flow, and that another such transition point exists, denoted by Re_{FT} ($Re_{FT} = 100$), at which the flow abruptly changes from Forchheimer to turbulent flow. The aforestated assumption permits the characterization of the entire flow range of interest using equations (4) and (5).

When an object, such as a cylinder, is embedded in a porous medium the porosity, ε, of the medium varies near the surface of the object—this is known as the 'wall effect'. For porous media whose matrices are composed of randomly packed spheres, ε varies from 0.36 far from a wall to unity at the wall. The rate of heat transfer from a heated object embedded in a porous medium is influenced by the wall effect. In their study of natural convection heat transfer from a horizontal cylinder embedded in a porous medium, Fand *et al.* [2] took the wall effect into account by replacing ε where it appears in the following definition of the effective thermal conductivity of the porous medium

$$k = \varepsilon k_f + (1-\varepsilon)k_s \tag{6}$$

by the so-called 'wall corrected porosity', ε_w, defined as follows:

$$\varepsilon_w = \varepsilon\left[1+\frac{1}{2}\left(\frac{d}{D}\right)^3\right] \tag{7}$$

where d/D is the ratio of the (spherical) particle diameter to the test cylinder diameter. The applicability of equation (7) was established in ref. [2] for $d/D \leqslant 0.35$.

The values of k_s in the present case (soda lime glass) may be calculated by the following equation:

$$k_s = 1.00416 + 1.6736\times10^{-3}T - 4.184\times10^{-6}T^2 \tag{8}$$

where T is the temperature in °C and k_s is expressed in W m^{-1} $°C^{-1}$. The values of k_f in the present case are those for water. The wall correction modifies both the Nusselt and Prandtl numbers via the effective thermal conductivity and it modifies the Grashof number via the permeability.

A phenomenon that occurs in all flows through porous media, and which must be considered in the present investigation, is 'dispersion'. The meaning of the term dispersion can be explained qualitatively by comparing the laminar one-dimensional flow of a fluid through a region of space in the presence of, and in the absence of, a porous matrix. In the absence of a porous matrix, the paths of all fluid particles are straight, parallel lines; whereas, in the presence of a porous matrix, each fluid particle follows a non-rectilinear path through the interstices of the porous medium. The trajectory of each fluid particle in a porous medium is a stochastic process, the result of which is an overall migration, or 'dispersion' of the particles away from the straight, parallel lines they would have followed in the absence of the porous medium. Dispersion can be quantified in terms of a 'dispersion coefficient' which has been shown by several authors to be a power function of Re_d.

EXPERIMENTAL APPARATUS AND PROCEDURE

The apparatus used in the present study consisted of the water tunnel and heated test cylinder described in ref. [1] plus a matrix of glass spheres randomly packed into the test section of the tunnel around the heated cylinder. Figure 1 shows the geometry of the tunnel's test section and its contents. The function of the perforated plates shown in Fig. 1 was to confine the glass spheres and yet allow a through-flow of water. The function of the screens shown in Fig. 1 interposed between the glass spheres and the perforated plates was to prevent clogging of the perforations by the glass spheres. A thermocouple placed just upstream of the test section was used to measure the bulk temperature of the flowing water. The velocity of the flow was controlled by means of a variable speed pump and measured by means of a series of calibrated orifices with which the tunnel is equipped. The velocity of flow in the test section was corrected for blockage by the cylinder in the same way as was done in ref. [1].

The porous matrix used in this study consisted of randomly and tightly packed soda lime glass spheres having a nominal diameter of 3 mm. In order to determine the diameter of the spheres precisely, one hundred spheres were weighed individually on a precision balance and, having determined the density of the glass, the mean equivalent diameter of the glass spheres was calculated and found to equal 3.072 mm. The porosity, ε, of the porous medium was determined experimentally to be 0.3606.

The heated test cylinder (diameter, D = 11.45 mm) contains a central electrical heating coil which was fed by a stable variable direct current power supply. A high precision wattmeter was used to measure the power dissipated in the test cylinder. A pair of independently powered guard heaters which flanked the central heater coil prevented axial heat transfer by conduction from the central heater. A pair of thermocouples located beneath the surface of the test cylinder was used to determine the surface temperature of the heated cylinder as described in ref. [1].

The experimental data consisted of simultaneous measurements of the surface temperature of the test cylinder and the bulk temperature of the water corresponding to arbitrarily imposed values of velocity in the tunnel and electrical power dissipation in the test heater. The ranges of the experimental variables used to determine the correlation equations presented below are as follows:

$$2 \leqslant q(\mathrm{kW\,m^{-2}}) \leqslant 327$$

$$19 \leqslant T_b(°\mathrm{C}) \leqslant 27$$

$$2.5 \leqslant \Delta T(°\mathrm{C}) \leqslant 61$$

$$0.034\times10^{-3} \leqslant v(\mathrm{m\,s^{-1}}) \leqslant 0.077$$

$$0.13 \leqslant Re_d \leqslant 344$$

$$0.5 \leqslant Re_D \leqslant 1280.$$

It is estimated that experimental errors in the present measurements of temperature, heat flux and velocity would result in errors not exceeding 3% in the Nusselt and Reynolds numbers. Additional errors in the determination of the Nusselt number were

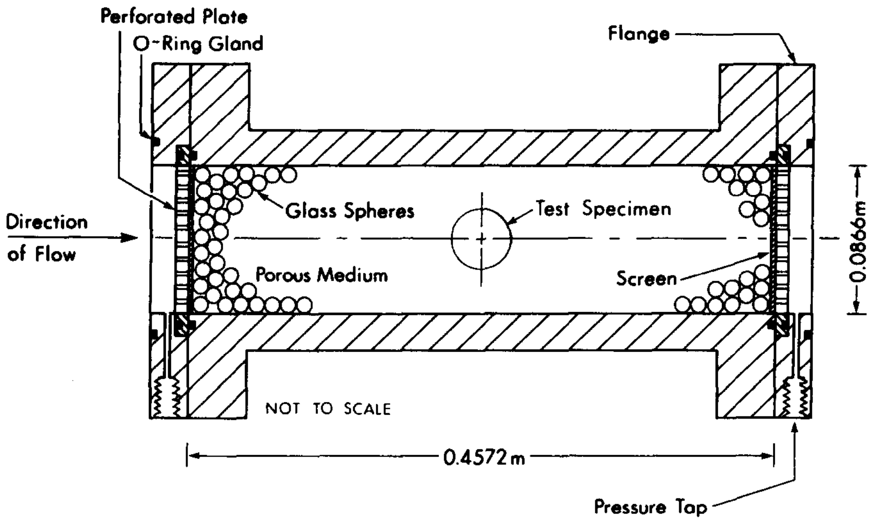

FIG. 1. Test section of water tunnel.

incurred due to the presence of air bubbles in the porous medium. Experience suggests that the presence of air bubbles can affect the present measured values of the Nusselt number by as much as 5%.

In the course of this investigation the test section of the water tunnel was emptied and refilled with (the same amount) of glass spheres several times. This was done in order to ascertain whether the experimental measurements would change significantly if the packing, which alters the 'microstructure' of the porous medium in the vicinity of the test specimen, is changed. It was found that the Nusselt number varied by about 5% about a mean in response to changes in packing with no change in porosity. Thus, a maximum overall experimental error in an individual measurement of the Nusselt number of 13%, or even 15%, is not unlikely.

METHOD OF DATA ANALYSIS

An empirical correlation of experimental data is developed by adopting an appropriate 'hypothesis', which usually consists of an algebraic equation containing arbitrary constants, and then determining the numerical values of the constants by fitting the hypothesis to the experimental data. A rational search for an appropriate hypothesis can follow any one or a combination of the following three routes: (1) utilization of clues provided by the results of a more-or-less approximate theoretical analysis (in which case the resulting correlation is called 'semi-empirical'; (2) adoption of forms which previous experience has demonstrated 'work' in analogous situations; (3) adoption of a form based upon physical reasoning but without the advantages of possessing the results of a theoretical analysis or prior experience in an analogous case. Since no theoretical solution to the present general problem is available, the method followed in the present study to determine a correlation hypothesis was a combination of procedures (2) and (3) above.

The correlation hypotheses adopted in the present investigation are based upon four plausible assumptions concerning the physical processes that occur in the problem under consideration. The first assumption (A1) is that cross flow past a heated cylinder embedded in a porous medium can be decomposed into two components, a 'coarse' component and a 'fine' component. The coarse component is a flow that has (conceptually) a velocity at every point in the field; thus, from the 'coarse' point of view, the porous matrix is 'transparent' to the flow. (Transparency is implicit in the well-known 'volume averaging' method of analyzing flows through porous media.) The coarse flow can be visualized as being that flow which would occur around a heated cylinder if the cylinder were immersed in a hypothetical fluid whose thermal conductivity was equal to the (wall-corrected) thermal conductivity of the porous medium and all of whose other thermophysical properties were the same as those of the fluid which saturates the porous medium. The fine component refers to the meandering flow through the interstitial spaces between the particles of the porous medium. The second assumption (A2) is that the influence of the coarse component of the flow in a porous medium upon the Nusselt number can be represented by analogous equations—such as, for example, equation (2) in the case of forced convection—if the thermal conductivity is taken to be the (wall corrected) equivalent thermal conductivity of the porous medium. The third assumption (A3) is that the fine component of flow will influence the diffusion of momentum near a heated surface and hence will change the numerical value of the exponent of the Prandtl number in the analogous equation. The fourth assumption (A4) is that the fine component of flow will, via dispersion, modify the dependence of the Nusselt number upon the Reynolds number and that this modification can be represented by multiplying the analogous equation by a factor which is proportional to a power function of Re_d. Taken together, these four assumptions lead to the following correlation hypothesis for forced convection:

$$Nu = [(0.255+0.699 Re_D^{0.5}) Pr^{0.29}][Pr^x][C\, Re_d^y] \quad (9)$$

where the brackets represent, successively, (A2), (A3) and (A4); and C, x and y are constants. Again, the equivalent wall-corrected thermal conductivity is to be used in equation (9).

A criterion for discriminating between those experimental data for which natural convection effects were negligible or not was established and applied. The (predominantly) forced convection data which fell into the Darcy, Forchheimer and turbulent regimes were correlated empirically using equation (9) by determining the optimum values of the constants C, x and y based upon the experimental data and then demonstrating that the errors incurred by the correlations so determined *vis-à-vis* the data are within acceptable bounds. These correlations were then shown to adequately represent the two intervening transition regimes by employing the assumption (see above) of the existence of (fictitious) transition points at $Re_{DF} = 3$ and $Re_{FT} = 100$.

The general method described above was also used to correlate those data for which natural convection effects are not negligible in comparison with forced convection effects. This procedure is described in detail below.

CORRELATION EQUATIONS

Forced convection: Darcy flow

Well over 100 experimental data points were obtained in the Darcy regime ($Re_d \leqslant 2.3$). It was obvious that natural convection effects would not be negligible for some of these points, namely, for those corresponding to relatively high ΔT and low velocity; and hence it was necessary to eliminate such points from consideration in the present endeavor.

In order to establish a quantitative criterion whereby irrelevant data points (i.e. data points for which natural convection effects are not negligible) could be identified and eliminated from consideration, the following procedure was adopted. First, it was assumed, by analogy with equation (1), that $Gr_K\, Re_D^{-2}$ is a quantity which provides a measure of the relative importance of natural convection to forced convection for heat transfer in porous media. This assumption implies that the 'coarse' component of flow discussed above provides an adequate measure of the relative importance of natural and forced convection. Since the magnitude of the upper bound on $Gr_K\, Re_D^{-2}$ whereby a judgement can be rendered as to whether natural convection is negligible in a particular case was not known, the upper bound was stated in terms of an unknown constant; thus, the criterion becomes

$$Gr_K\, Re_D^{-2} \leqslant \phi \quad (10)$$

where ϕ is a constant yet to be determined. Next, the value of ϕ was arbitrarily chosen equal to 4. This choice is purposely too lax, that is, it does not eliminate points from consideration which *should* be eliminated. One hundred and eleven experimental data points that fell in the Darcy regime satisfied the condition $Gr_D\, Re_D^{-2} \leqslant \phi = 4$. A computerized regression procedure was then used to calculate the values of C, x and y in equation (9) corresponding to minimum E_{rms} for these 111 points. The values of $|E_{max}|$ and E_{md} were also calculated and are listed in Table 1.

Table 1. Errors incurred by equation (9) corresponding to different values of ϕ for $Re_d \leqslant 2.3$

ϕ	N	$\%E_{md}$	$\%\lvert E_{max}\rvert$
0.2	39	5.51	11.1
0.3	48	5.52	11.8
0.4	54	5.64	11.5
0.5	60	5.42	11.4
0.6	63	5.55	11.2
0.7	67	5.52	12.2
0.8	71	5.65	12.5
0.9	77	5.67	13.1
1.0	78	5.63	13.2
1.5	90	6.07	15.3
2.0	99	6.36	17.4
4.0	111	7.93	22.8

Table 1 shows that for $\phi = 4$, the errors incurred by equation (9), in particular $|E_{max}|$, are excessive compared with the overall experimental error in the measurement of the Nusselt number. This was expected, inasmuch as equation (9) purports to represent *forced convection*, whereas some of the data points considered when $\phi = 4$ contain non-negligible *natural convection* effects. Having verified this anticipated result, the value of ϕ was then decreased in stages and the correlation procedure using equation (9) was repeated with the results listed in Table 1.

Table 1 shows that when $\phi \leqslant 0.6$, the errors remain nearly constant and are consistent with the magnitude of the overall experimental error in the measurement of the Nusselt number. In consideration of equation (1), the *conservative* value $\phi = 0.5$ was adopted, and with it the criterion embodied in equation (10) becomes

$$Gr_K\, Re_D^{-2} \leqslant 0.5. \quad (11)$$

The preceding unbiased analysis of the data leads to a value of ϕ very nearly identical to the number on the right-hand side of equation (1). This constitutes empirical evidence of the validity of the assumption that an analogy exists between the present case and the case corresponding to equation (1), and that the coarse component of the flow provides an adequate measure of the relative importance of natural and forced convection. It is relevant to mention here that

Table 2. Errors incurred by equations (12)–(15) in applicable ranges of Re_d

Range	Equation	N	$E_{rms}\times 10^3$	$\%E_{md}$	$\%\lvert E_{max}\rvert$
$Re_d \leqslant Re_{DH}$	12	60	8.21	5.42	11.4
$Re_{DH} < Re_d \leqslant Re_{DF}$	12	28	15.9	7.35	15.2
$Re_{DF} < Re_d \leqslant Re_{FL}$	13	57	10.5	7.09	16.9
$Re_{FL} < Re_d \leqslant Re_{FH}$	13	438	1.80	2.95	12.8
$Re_{FH} < Re_d \leqslant Re_{FT}$	13	46	8.42	3.59	15.7
$Re_{FT} < Re_d \leqslant Re_{TL}$	14	37	6.72	2.31	13.2
$Re_d > Re_{TL}$	14	168	2.75	2.53	15.0
$Re_d \leqslant Re_{DH}$	15	51	8.22	4.74	13.7

$Re_{DH} = 2.3$, $Re_{DF} = 3.0$, $Re_{FL} = 5.0$, $Re_{FH} = 80$, $Re_{FT} = 100$, $Re_{TL} = 120$.

Cheng [4] determined analytically an analogous criterion, $Gr_K\,Re_D < 0.15$, for aiding flow. This criterion was found to be overly restrictive when applied to the present case.

The optimum values of C, x and y corresponding to $Gr_K\,Re_D^{-2} \leqslant 0.5$ in the Darcy regime are 2.17, -0.102 and 0.230, respectively. Thus, equation (9) yields the following correlation for forced convection heat transfer from a cylinder embedded in a porous medium with cross flow in the Darcy regime:

$$Nu = 2.17(0.255+0.699Re_D^{0.5})Pr^{0.188}\,Re_d^{0.230}. \quad (12)$$

The errors incurred by equation (12) *vis-à-vis* the experimental data are listed in Table 2.

Forced convection: Forchheimer and turbulent flows

None of the experimental data points which fell in the Forchheimer or turbulent regimes were eliminated by the criterion embodied in equation (11). The optimum values of the constants C, x and y for the Forchheimer regime were 2.15, -0.136 and 0.126, respectively; and for the turbulent regime they were 1.48, 0 and 0.179, respectively. These values of the constants result in the following correlation equation for Forchheimer flow:

$$Nu = 2.15(0.255+0.699Re_D^{0.5})Pr^{0.154}\,Re_d^{0.126}. \quad (13)$$

and for turbulent flow:

$$Nu = 1.48(0.255+0.699Re_D^{0.5})Pr^{0.290}\,Re_d^{0.179}. \quad (14)$$

The errors incurred by these correlation equations *vis-à-vis* the experimental data are listed in Table 2. It is interesting to note that the result $x = 0$ for turbulent flow implies that in this regime the 'fine' component of the flow does not affect the exponent on Pr that is determined (conceptually) by the 'coarse' component of the flow.

Forced convection: transition regimes

The errors incurred by using equation (12) to represent the data from $Re_{DH} = 2.3$ to $Re_{DF} = 3$ and by using equation (13) to represent the data from $Re_{DF} = 3$ to $Re_{FL} = 5$; and the errors incurred by using equation (13) to represent the data from $Re_{FH} = 80$ to $Re_{FT} = 100$ and by using equation (14) to represent the data from $Re_{FT} = 100$ to $Re_{FL} = 120$ are all listed in Table 2.† These errors are not excessive and hence it follows that equations (12)–(14) can be used to calculate the Nusselt number for forced convection over the entire range of interest of the Reynolds number.

† *Nota bene* that $2.3 < Re_d \leqslant 5$ comprises the transition region from Darcy to Forchheimer flow and that $80 < Re_d \leqslant 120$ comprises the transition region from Forchheimer to turbulent flow.

Combined forced and natural convection: Darcy flow

Fifty one data points were obtained in the range $0.5 < Gr_K\,Re_D^{-2} \leqslant 4$, for which natural convection effects are significant. All of these data fell in the Darcy regime ($Re_d \leqslant 2.3$). The basic correlation hypothesis adopted in ref. [1] was employed here to correlate the present combined convection data. Thus, a natural convection component, per equation (3), was added to the forced convection expression, per equation (12), to yield the following combined correlation equation:

$$Nu = [2.17(0.255+0.699Re_D^{0.5})+C(Gr_K\,Re_D^{-2}) \times (Gr_K^{0.649}\,Pr^{0.525})]Pr^{0.188}\,Re_d^{0.230} \quad (15)$$

where C is a constant to be determined from the data. It was found that the optimum value for C is 0.0451. With this value for C, equation (15) yields the errors shown in Table 2 *vis-à-vis* the experimental data. These errors are consistent with the experimental error. Equation (15) collapses to equation (12) as $Gr_K\,Re_D^{-2}$ approaches zero.

RESUME AND CONCLUSIONS

It has been shown that correlation hypotheses have been determined based upon a set of four assumptions, paramount among which is the assumption that cross flow past a heated cylinder embedded in a porous medium can be decomposed conceptually into 'coarse' and 'fine' components, which have lead to the successful correlation of an extensive set of convection heat transfer data. The data extends from the Darcy regime into the turbulent regime and spans the intervening Forchheimer and transition regimes. A criterion, namely $Gr_K\,Re_D^{-2} \leqslant 0.5$, has been established which indicates whether natural convection effects are negligible in a given situation. For the present geometry and for $Gr_K Re_D^{-2} \leqslant 0.5$ the following three correlation equations have been determined:

$$Nu = 2.17(0.255+0.699Re_D^{0.5}) \times Pr^{0.188} Re_d^{0.230}; \quad 0.5 < Re_d \leqslant 3 \quad (16)$$

$$Nu = 2.15(0.255+0.699Re_D^{0.5}) \times Pr^{0.154} Re_d^{0.126}; \quad 3 < Re_d \leqslant 100 \quad (17)$$

$$Nu = 1.48(0.255+0.699Re_D^{0.5}) \times Pr^{0.290} Re_d^{0.179}; \quad Re_d > 100. \quad (18)$$

For $0.5 < Gr_K Re_D^{-2} \leqslant 4$ the following correlation equation has been determined for Darcy flow ($0.1 < Re_d < 2.3$):

$$Nu = [2.17(0.255+0.699Re_D^{0.5}) + 0.0451(Gr_K Re_D^{-2})(Gr_K^{0.649} Pr^{0.525})] \times Pr^{0.188} Re_d^{0.230}. \quad (19)$$

All properties in equations (16)–(19) are to be evaluated at the mean film temperature. Since the applicability of equation (7) was established in ref. [2] for $d/D \leqslant 0.35$, this must be regarded as a restriction upon equations (16)–(19) pending further information.

In conclusion, it is suggested that the correlation procedure adopted here may yield useful results if applied to other geometries, such as, for example, forced convection heat transfer in ducts packed with porous media.

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CONVECTION THERMIQUE AUTOUR D'UN CYLINDRE HORIZONTAL NOYE DANS UN MILIEU POREUX

Résumé—On présente les résultats d'une étude expérimentale de transfert de chaleur par convection mixte autour d'un cylindre horizontal noyé dans un milieu poreux composé de sphères de verre, rangées au hasard, et saturé d'eau. La direction de l'écoulement d'eau est horizontale et normale à l'axe longitudinal du cylindre. Le diamètre du cylindre D est 11,45 mm et le diamètre équivalent des billes de verre est 3,072 mm. On montre que la condition $Gr_K/Re_D^2 \leqq 0,5$ représente un critère conservatif pour séparer les données qui sont fortement gouvernées par la convection forcée de celles pour lesquelles les effets de convection naturelle sont forts. On présente un explication basée sur quatre hypothèses dont la principale est que l'écoulement peut être considéré comme étant représentable par une composante 'grossière' et une autre 'fine'. Cette hypothèse fournit une base pour unifier un ensemble de données de transfert de chaleur qui s'étend depuis le régime de Darcy jusqu'aux régimes de Forchheimer et de transition. On suggère que la procédure d'analyse adoptée peut être appliquée favourablement à d'autres géométries comme, par exemple, la convection thermique forcée dans les canaux remplis par un milieu poreux.

WÄRMEÜBERGANG BEI DER KOMBINATION VON ERZWUNGENER UND NATÜRLICHER KONVEKTION VON EINEM IN EINEM PORÖSEN MEDIUM EINGEBETTETEN HORIZONTALEN ZYLINDER

Zusammenfassung—Diese Arbeit berichtet über die Ergebnisse einer experimentellen Untersuchung des Wärmeübergangs bei der gemischten erzwungenen und natürlichen Konvektion von einem horizontalen Zylinder. Dieser ist in einem porösen Medium eingebettet, das aus zufällig angeordneten Glaskugeln besteht und mit Wasser gesättigt ist. Die Fließrichtung des Wassers war horizontal und normal zur Längsachse des Zylinders. Der Durchmesser des Zylinders war $D = 11{,}45$ mm und der äquivalente Durchmesser der Glaskugeln 3,072 mm. Es wird gezeigt, daß die Bedingung $Gr_K/Re_D^2 = 0,5$ ein konservatives Kriterium darstellt, das Wärmeübergangsdaten, die überwiegend von erzwungener Konvektion beherrscht werden, von denen unterscheidet, wo die natürliche Konvektion bedeutend ist. Eine Korrelationshypothese für konvektiven Wärmeübergang wird vorgestellt, die auf vier Annahmen basiert, von denen die wichtigste ist, daß der Durchfluß (begrifflich) als zusammengesetzt aus "groben" und "feinen" Komponenten angesehen werden kann. Diese Hypothese verschafft eine Grundlage für die erfolgreiche Korrelation eines Satzes experimenteller Wärmeübergangsdaten, der sich vom Darcy-Bereich bis zum turbulenten Bereich erstreckt und den dazwischenliegenden Forchheimer-Bereich und das Übergangsgebiet überspannt. Es läßt sich vermuten, daß das hier angenommene Korrelationsverfahren brauchbare Ergebnisse liefert, wenn es auf andere Geometrien angewendet wird, z. B. auf den Wärmeübergang bei erzwungener Konvektion in mit porösem Medium gefüllten Kanälen.

КОМБИНИРОВАННЫЙ ВЫНУЖДЕННЫЙ И ЕСТЕСТВЕННОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС ОТ ГОРИЗОНТАЛЬНОГО ЦИЛИНДРА, ПОГРУЖЕННОГО В ПОРИСТУЮ СРЕДУ

Аннотация—Представлены результаты экспериментального исследования теплопереноса при смешанной конвекции от горизонтального цилиндра, погруженного в пористую среду, которая состоит из хаотически расположенных стеклянных шариков, насыщенную водой. Поток воды двигался горизонтально и перпендикулярно оси цилиндра. Диаметр цилиндра D составлял 11,45 мм, а эквивалентный диаметр стеклянного шарика был равен 3,072 мм. Показано, что условие $Gr_K Re_D \leqslant 0{,}5$ характеризует область отклонения данных по теплопереносу, которые определяются преимущественно вынужденной конвекцией, от тех, в которых вклад естественной конвекции значителен. Предложена методика аппроксимации данных по конвективному теплопереносу, которая основана на четырех предположениях, основное из которых состоит в том, что течение может считаться состоящим из «грубых» и «тонких» компонентов. Показано, что эта методика создает основу для успешной корреляции набора экспериментальных данных по теплообмену, которая охватывает диапазон от режима, описываемого законом Дарси, до турбулентного, а также промежуточный режим Форшхаймера и переходный. Предполагается, что предложенная методика обобщения может дать полезные результаты, если применять ее к таким геометриям, например, как вынужденный конвективный теплоперенос в каналах, заполненных пористыми средами.